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Intersecting branes and adding flavors to the Maldacena-Nŭnez background

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Abstract: Following a proposal in ref. [3], we study adding flavors into the Maldacena-Nŭnez background. It is achieved by introducing spacetime filling D9-branes or intersecting D5′-branes into the background with a wrapping D5-brane. Both D9-branes and D5′-branes can be spacetime filling from the 5D bulk point of view. At the probe limit it corresponds to introducing non-chiral fundamental flavors into the dual $\mathcal{N} = 1$ SYM. We propose a method to twist the fundamental flavor which has to involve open string charge. It reflects the fact that coupling fundamental matter to SYM in the dual string theory corresponds to adding an open string sector.

Keywords: D-branes, String Duality, AdS-CFT and dS-CFT Correspondence, Supersymmetry and Duality.

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Contents

1. Introduction

It is expected that 4d pure Yang-Mills theory, with gluon as degrees of freedom, is mapped to an entirely 5d non-critical closed string theory background [1]. This expectation has been realized in many supersymmetric examples starting from a proposal on AdS/CFT correspondence [2]. In these examples we knew the geometry of the background. Although it is difficult to quantize string theory on the background, it was conjectured that at the large-N limit, the super Yang-Mills theories (SYM) are dual of the low energy effective theories of a closed string theory, i.e., supergravity (SUGRA), on various backgrounds. When we add massless fundamental flavors to SYM, however, it is very difficult to get the dual background. Adding fundamental flavors effectively introduces an open string sector to the string theory. Light open string states inevitably modify the original closed string background. In other words, D-branes, which generate non-trivial background, have to carry open string charge besides the closed string charge (R-R charge).

It was noted by Karch and Katz [3] that this difficulty can be partially overcome at certain probe limit. They proposed an effective method to add flavors into AdS/CFT correspondence via introducing few D7-brane probe into $AdS_5 \times S^5$. The key points of their proposal can be summarized as follows:

- Adding fundamental flavors in the gauge theory is equivalent to adding space-time filling D-branes into the 5d bulk theory.

- It is not necessary to introduce orientifold in order to cancel the R-R tadpole of space-time filling D-branes. Space-time filling D-branes can wrap on a topological trivial cycle inside the original background. This way they may not carry any charge and avoid a R-R tadpole.
- The stability of these space-time filling D-branes is ensured by the negative mass modes in the AdS space, but the mass is above the BF bound [4, 5] and they do not lead to an instability in the curved 5d geometry.
- At the probe limit, the effect of probe D-branes on the bulk geometry can be ignored. The bulk geometry is still AdS but the dual field theory is now CFT with a defect or boundary (dCFT). When fundamental flavors get mass, it corresponds to bulk geometry receiving supergravity fluctuations.

It is interesting to extend their proposal because it provides a possible way to add fundamental flavors in gauge/string duality on known backgrounds. Authors of ref. [6] has incorporated massive fundamental quarks in supergravity dual of $\mathcal{N}=1$ SYM by introducing D7 brane probes in Klebanov-Strassler background. In ref. [7] it was shown how to realize chiral symmetry breaking and production of pseudoscalar mesons in some non-supersymmetric gauge/gravity duals. The purpose of the present paper is to consider adding fundamental flavors into $\mathcal{N}=1$ SYM which is dual to the well-known Maldacena-Nŭnez (MN) background [8].

The MN background is described by N D5-branes wrapping on a topological non-trivial 2-cycle inside a CY three-fold. The normal bundle within the CY space has to be twisted in order to preserve some supersymmetry. This configuration is much complicated than the one considered by Karch and Katz, in which the D3-branes background are flat. Similar to adding D7-branes into the D3-brane background in the AdS case, we may consider to add finite number of D9-branes or D5-branes into the MN background at the large N limit respectively. In the following we shall discuss two different cases respectively.

(a) Adding M D9-branes: in the flat case, D5-D9 system supports $D=6$, $\mathcal{N}=1$ supersymmetry. Adding D9-branes is effectively to add M flavor hypermultiplets into the 6d SU(N) gauge theory. When D5-branes wrap on a supersymmetric 2-cycle, the 6d $\mathcal{N}=2$ vector multiplet on the world-volume of D5-branes reduces to 4d $\mathcal{N}=2$ or $\mathcal{N}=1$ vector multiplet. The background geometry is $\mathbb{R}^4\times CY_3$. Therefore, D9-branes have to wrap on a CY three-fold so that supersymmetry is still kept. This configuration, however, seems to receive some unexpected features. The first problem should be asked whether D9-branes can be treated as a probe since CY three-fold is open. If it is not true, MN solution is not a well-defined background to describe this system. The second problem is that the D9-branes shall always carry induced R-R charge from the background, whether they are treated as probe or not. This is different from the case discussed in [3]. At rigid probe limit the effects of R-R tadpole can be ignored. The original MN background is still good description on this configuration. When we study this system beyond probe limit, we encounter dangerous

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R-R tadpoles. They have to be cancelled by either introducing orientifold planes or anti-D9s. The latter is in general unstable, and the former destroy MN background description.

Although there are those unpleasant problems, it is helpful to count spectrum of the dual field theory. The twist on vector multiplet by D5-branes is well-known. The twist on hypermultiplet, however, should be checked carefully. In general, the original twist on 6d hypermultiplet is no longer to leave any massless states, otherwise D9-branes can not be felt by the field theory. As discussed above, when massless fundamental flavors are introduced, D5-branes will carry open string charge. A natural suggestion is that these charges should play a role in twisting supermultiplet. We will show that it indeed works.

(b) Adding M D5$'$-branes: the D5$'$-branes intersect with the original 5-branes, and have four dimensions in common. In the flat case, this system supports a $D = 4$, $\mathcal{N} = 2$ $\mathrm{SU}(N) \times \mathrm{SU}(M)$ gauge theory. Adding D5$'$-branes is to add M flavor hypermultiplet into the 4d $\mathrm{SU}(N)$ gauge theory. When D5-branes wrap on a supersymmetric 2-cycle inside a CY 3-fold, the D5$'$-branes should wrap another orthogonal supersymmetric 2$'$-cycle inside the CY space. If the 2$'$-cycle is topological non-trivial, D5$'$-branes carry R-R charge from the background. The charge, however, does not bring dangerous tadpole problem since the CY space is open while the 2$'$-cycle is compact. If the 2$'$-cycle is topological trivial, D5$'$-branes do not get R-R charge and we don't have tadpole problem again. Thus adding D5$'$-branes is a nice choice: D5$'$-branes can be treated as a probe in the original D5-brane background and it does not cause the tadpole problem.

The twist on various supermultiplet, similar to the D5-D9 case, should be checked carefully. The open string charge will play a role in twisting in the hypermultiplet.

When D5$'$-branes does not carry induced charge from the background, stability of D-branes should be considered. The stability of D-branes is ensured by the similar mechanism in [3, 9, 10]: there are negative mass modes in 5d world which control the slipping of the D-branes off the cycle they wrap, but the mass is not negative enough to lead to an instability in the curved 5d geometry. We will show that similar mechanism works in our cases. The negative mass modes in 5d bulk spacetime are generated by Kaluza-Klein spectrum on a compact 5-cycle inside a CY three-fold, and have a lower bound (similar to BF bound in AdS space).

The paper is organized as follows: in section 2 we consider adding D9-branes to the MN background. By a careful twist, the flat part of the system supports $\mathcal{N} = 1$ SYM with fundamental matter. In section 3 we show how to add D5$'$-branes to the MN background and twist it. The flat part of this system again supports $\mathcal{N} = 1$ SYM with fundamental matter. We give a brief conclusion in section 4. The appendix of this paper devotes to computing KK modes on the compact part of CY three-fold which is described by the MN solution.

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2. Adding D9-branes into the wrapped D5-brane background

2.1 The flat spectrum

The massless spectrum for flat D5-D9 system has been given in ref. [11]. In the following we review some details. Assuming D5-branes spread in {0, 1, 2, 3, 4, 5} directions, the massless spectrum of this system is as follows,

- 5-5 open strings generate $D = 6$, $(1,1)$ vector supermultiplet with R-symmetry $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.
- 9-9 open strings generate $D = 10$, $\mathcal{N} = 1$ vector supermultiplet.
- 5-9 open strings: in the NS-sector four periodic world-sheet fermions ψ^i, namely in ND directions $i = 6, 7, 8, 9$, generate four massless bosonic states. We label their spins in the (6,7) and (8,9) planes, $|s_3, s_4\rangle$ with s_3, s_4 taking values $\pm\frac{1}{2}$. The GSO projection requires $s_3 = s_4$ so that two massless bosons survive. In the R-sector four transverse world-sheet fermions ψ^m with $m = 2, 3, 4, 5$ are periodic. They again generate four massless fermionic states, and are labelled by $|s_1, s_2\rangle$. The GSO projection requires $s_1 = -s_2 = \pm\frac{1}{2}$ so that again two massless fermions survive. Then massless content of the 5-9 spectrum amounts to a $D = 6$ half-hypermultiplet. The other half comes from strings of opposite orientation, 9-5 strings. From the point of view of D9-brane world volume, 5-9 and 9-5 carry opposite charges. In other words, they associate to a global $\mathrm{U}(1)_o$ symmetry, where "o" denotes orientation.[1] For the sake of convenience we split 10d Lorentz group into

$$\begin{aligned} \mathrm{SO}(1,9) &\longrightarrow \mathrm{SO}(1,1) \times \mathrm{SO}(4)_N \times \mathrm{SO}(4)_D \\ &\simeq \mathrm{SO}(1,1) \times \mathrm{SU}(2)_{\mathrm{NL}} \times \mathrm{SU}(2)_{\mathrm{NR}} \times \mathrm{SU}(2)_{\mathrm{DL}} \times \mathrm{SU}(2)_{\mathrm{DR}} \,. \end{aligned} \tag{2.1}$$

  The massless states lie in the following representation of the group

$$\mathrm{SO}(1,1) \times \mathrm{SU}(2)_{\mathrm{NL}} \times \mathrm{SU}(2)_{\mathrm{NR}} \times \mathrm{SU}(2)_{\mathrm{DL}} \times \mathrm{SU}(2)_{\mathrm{DR}} \times \mathrm{U}(1)_o \,,$$

  – bosons: $(0,\ 1,\ 1,\ 2,\ 1,\ \pm)$
  – fermions: $(\pm 1/2,\ 1,\ 2',\ 1,\ 1,\ \pm)$.

2.2 Wrapping D5-branes on a 2-cycle

Wrapping world-volume of N D5-branes on a topological non-trivial 2-cycle leads to a non-supersymmetric 3-branes. In order to preserve supersymmetry one need to twist the normal bundle of this 2-cycle, i.e., to identify U(1) charge on a 2-cycle (denotes by $\mathrm{U}(1)_J$) with an external gauge field. For D5-branes there are two choices: to identify the $\mathrm{U}(1)_J$ charge with the charge of diagonal sub-$\mathrm{U}(1)_D$ group of R-symmetry group $\mathrm{SU}(2)_{\mathrm{DL}} \times \mathrm{SU}(2)_{\mathrm{DR}}$, or with the charge of subgroup $\mathrm{U}(1)_{\mathrm{DL}}(\mathrm{U}(1)_{\mathrm{DR}})$ of $\mathrm{SU}(2)_{\mathrm{DL}}(\mathrm{SU}(2)_{\mathrm{DR}})$. Some massless

[1] This global $\mathrm{U}(1)_o$ symmetry, however, is U(1) gauge symmetry on D9 world-volume from view point of world-volume field on D9-branes.

states become massive under the twist and decouple with other massless states. The flat part of D5-branes supports a (3+1)-dimensional twist gauge theory with $\mathcal{N}=2$ or $\mathcal{N}=1$ supersymmetry corresponding to the previous two choices.

When M D9-branes are introduced, the twist on vector supermultiplet on D5-branes is the same as the case without D9-branes. The key point is to focus on twisting hypermultiplet generated by 5-9 and 9-5 string. When an D5-brane wrap on S^2, the 6d hypermultiplet in the previous subsection reduces to:

Bosons: $(2,1,\pm)\times|0,0\rangle_0$.

Fermions: $(1,1,\pm)\times(|\pm 1/2,1/2\rangle_-\oplus|\pm 1/2,-1/2\rangle_+)$,

where $|s_0,s_1\rangle$ is the spin basis in 4d spacetime, subscript "0, ±" denotes the $\mathrm{U}(1)_J$ charge, and (x, x, x) denotes the representation of $\mathrm{SU}(2)_{\mathrm{DL}}\times\mathrm{SU}(2)_{\mathrm{DR}}\times\mathrm{U}(1)_o$.

Now let us consider twisting on the above states. Naively we may consider two choices mentioned above:

1) $\mathrm{U}(1)_J=\mathrm{U}(1)_D\in D(\mathrm{SU}(2)_{\mathrm{DL}}\times\mathrm{SU}(2)_{\mathrm{DR}})$.

   Bosons: $2\times|0,0\rangle_\pm$.

   Fermions: $2\times|\pm 1/2,1/2\rangle_-\oplus 2\times|\pm 1/2,-1/2\rangle_+$.

   Here subscript "$\pm''$ denotes the total $\mathrm{U}(1)=\mathrm{U}(1)_J\times\mathrm{U}(1)_D$ charge. Hence there are no massless states surviving and the resulted field theory is still pure $D=4$, $\mathcal{N}=2$ SYM.

2) $\mathrm{U}(1)_J=\mathrm{U}(1)_L\in\mathrm{SU}(2)_{\mathrm{DL}}$. This choice is the same as the first one, but the resulted field theory is pure $D=4$, $\mathcal{N}=1$ SYM.

3) $\mathrm{U}(1)_J=\mathrm{U}(1)_R\in\mathrm{SU}(2)_{\mathrm{DR}}$. Denoting the total charge by $\mathrm{U}(1)=\mathrm{U}(1)_J\times\mathrm{U}(1)_R$ we have

   Bosons: $4\times|0,0\rangle_0$.

   Fermions: $2\times|\pm 1/2,1/2\rangle_-\oplus 2\times|\pm 1/2,-1/2\rangle_+$.

   Then this choice breaks supersymmetry.

We see that all of the above choices do not generate massless fundamental matters in the framework of 4d supersymmetric gauge theory. It can be understood naturally: the fundamental matter carries open string charges. The charges should play a role in twisting on fundamental matter. Consequently we can say that the 4d field theory gets effects from the open string sector. Therefore, we propose the following twist,

1) $\mathrm{U}(1)_J=\mathrm{U}(1)_D\in D(\mathrm{SU}(2)_{\mathrm{DL}}\times\mathrm{SU}(2)_{\mathrm{DR}})=\mathrm{U}(1)_o$. The total charge is of $\mathrm{U}(1)=\mathrm{U}(1)_J\times\mathrm{U}(1)_D\times\mathrm{U}(1)_o$.

   Bosons: $2\times|0,0\rangle_0\oplus|0,0\rangle_{++}\oplus|0,0\rangle_{--}$.

   Fermions: $|\pm 1/2,\pm 1/2\rangle_0\oplus|\pm 1/2,1/2\rangle_{--}\oplus|\pm 1/2,-1/2\rangle_{++}$.

Two real scalars (one complex scalar) and one Majorana spinor survive under the twist. They form a chiral multiplet of $D = 4$, $\mathcal{N} = 1$ superalgebra. Because the supermultiplet generated by 5-5 string does not carry $\mathrm{U}(1)_o$ charge, the twist on vector multiplet on D5-branes is not changed. Then we obtain $D = 4$, $\mathcal{N} = 1$ gauge theory which contains a vector multiplet, a chiral multiplet in the adjoint representation and M chiral multiplets in the fundamental representation of the gauge group.

2) $\mathrm{U}(1)_J = \mathrm{U}(1)_L \in \mathrm{SU}(2)_{\mathrm{DL}} = \mathrm{U}(1)_o$. Again two real scalars and one Majorana spinor survive under the twist. We obtain a $D = 4$, $\mathcal{N} = 1$ gauge theory which contains a vector multiplet and M chiral multiplets in the fundamental representation of the gauge group.

3) $\mathrm{U}(1)_J = \mathrm{U}(1)_R \in \mathrm{SU}(2)_{\mathrm{DR}} = \mathrm{U}(1)_o$. No massless states survive at this case.

Our proposal is indeed valid for 1) and 2). The $\mathrm{U}(1)_o$ charges play essential role in twisting on various string spectrum. Because the $\mathrm{U}(1)_o$ symmetry is gauge symmetry on D9-brane world-volume, it looks like that some world-volume field are turned on. This is true since endpoints of each 5-9 and 9-5 string pair carry opposite charges. Then a non-vanishing flux crosses on a pair of endpoints and lies within D9-brane worldvolume. The fluxes act as background gauge field strength F on world-volume. Similar to role of B-field background discussed in [12], the worldvolume F-field compensates breaking of supersymmetry when introducing D9-brane in wrapping D5-brane background. The same conclusion can be obtained from the following supergravity argument.

Because endpoints of an open string couple to NS 1-form potential, NS 1-form potential should be introduced into supergravity when we introduce fundamental flavor in the dual field theory. To make S^3 reduction, R-R 2-form potential induces SO(4) gauge field [13] in the 7d gauged supergravity while NS 1-form is still kept. Then the twist on normal bundle of the 2-cycle now becomes

$$\omega_M = A_M^{RR} + A_M^{NS}\,, \tag{2.2}$$

where ω_M is the spin connection of the 2-cycle. The connection on normal bundle is now cancelled by R-R field generated from the close string sector together with NS 1-form generated by the open string sector.

2.3 Discussions

Several open questions should be asked here. The first question is whether an D9-brane can be treated as a probe in the wrapped D5-brane background for $N \to \infty$, $M \sim$ fixed. If we treat an D9-brane as a probe, i.e., the effect of D9-branes is ignored, D9-branes wrap on the CY three-fold described by the wrapping D5-brane background. The energy of D9-branes, however, is still infinity since CY space is open. The same puzzle appears in introducing a D7-branes probe into a D3-brane background proposed by Karch and Katz [3]. An effective treatment is to introduce a cut-off near the boundary. Then energy of D7-branes will be of order to the energy of a D3-brane probe locating at the boundary of the background geometry. Consequently D7-branes can be treated as a probe as same as a D3-branes probe. For D9-branes, however, this treatment does not work well. For example, in the MN background the ratio of the energy of D9-branes to one of the D5-brane background locating at the boundary is of order $M \ln^4(r_0/l)/N$ where $r_0 \to \infty$ is a cut-off and l is the definite scale. Therefore, it is decided by fine tuning between M/N and $\ln^4(r_0/l)$ whether D9-branes can be treated as a probe.

Whether D9-branes are treated as probe or not, D9-branes carry R-R charge induced from wrapped D5-brane background. At rigid probe limit the analysis is fine. Beyond probe limit, however, we encounter unexpect R-R tadpole problem. This correction is order M/N, and is also implied by fundamental matter spectrum found in previous subsection that the spectrum is anomalous due to lack of anti-fundamental matter. The anomaly (or R-R tadpole) makes theories be inconsistent. They should be cancelled via introducing orientifold planes or anti-D9s. The former will destroy original wrapped D5-brane background even though D9-branes are treated as probe. The introduction on anti-D9-branes looks like strange since brane-anti-brane system is in general non-supersymmetric and unstable. This is not fact when background world-volume F-field is turned on. It was shown in refs. [14, 15] that, the brane-anti-brane system will be 1/4 BPS states when background magnetic fields take opposite directions on worldvolume of branes and anti-branes respectively. In addition, the specific worldvolume background electric field makes the usual tachyonic degrees disappear. Thus the system will be stable. Both of these conditions can be satisfied here since a non-vanishing background F-field is turned on D9-brane worldvolume. Then introduction on anti-D9s to cancel R-R tadpole is good choice. The open strings stretching between D5-branes and anti-D9s give massless anti-fundamental flavors which cancels anomaly in massless spectrum.

When D9-branes are treated as a probe, it is particularly interesting for the second choice on twist. For this twist supergravity on the MN background is dual to a (non-chrial) SU(N) SQCD with M fundamental flavors at the probe limit and the large-N limit. It reflects the fact that at the large-N limit the role of gluons is more important than the role of quarks (without chiral symmetry spontaneously breaking effects). The effect of fundamental flavors is reflected by DBI action on D9-brane worldvolume as in ref. [16, 17]. In order to manifest the quantum number of fundamental flavors, NS gauge field should be turned on in supergravity, i.e., we have to consider type-I string fluctuations on the type-IIB background. It corresponds to fluctuation on the MN background. The leading order correction is expected to be order of M/N but cancelled by anti-D9s.

3. Adding intersecting D5$'$-branes

We consider N ($N \to \infty$) D5-branes intersecting with M ($M \sim$ fixed) D5$'$-branes. The D5-branes spread in {0,1,2,3,4,5} directions while the D5$'$-branes spread in {0,1,2,3,6,7} directions. It is different from the D5-D9 system. We can naturally treat D5-branes as a background and D5$'$-brane as a probe on this background.

3.1 Flat spectrum

The flat spectrum on intersecting D5-branes was presented in ref. [18]. Here we list the main results for the sake of convenience as follows. Massless spectrum of 5-5 string and $5'$-$5'$ string are 6d (1,1) vector supermultiplet on D5-brane and D$5'$-brane worldvolume respectively. The new ingredient is the 5-$5'$ string. We divide 10d spacetime into three sectors: the NN sector with Newmann-Newmann boundary conditions for X^μ ($\mu = 0,1,2,3$), the DD sector with Dirichlet-Dirichlet boundary condition for X^m ($m = 8,9$) and the ND sector with Dirichlet-Newmann boundary condition for X^i ($i = 4,5,6,7$). In the NS sector, two real scalars survive by GSO projection, namely $|s_2, s_3\rangle$ with $s_2 = s_3 = \pm 1/2$ where s_2 and s_3 denotes spins in (4,5) and (6,7) planes. In the R sector, two fermions survive by GSO projection. That is $|s_1, s_4\rangle$ with $s_1 = -s_4 = \pm 1/2$, where s_1 and s_4 denotes spins in (2,3) and (8,9) planes respectively. Together with massless states generated by $5'$-5 string, they form a hypermultiplet of $D = 4$, $\mathcal{N} = 2$ superalgebra.

Ten-dimensional Lorentz symmetry now breaks down to

$$\mathrm{SO}(1,9) \longrightarrow \mathrm{SO}(1,3) \times \mathrm{SO}(2)_{\mathrm{ND}} \times \mathrm{SO}(2)_{\mathrm{ND}'} \times \mathrm{SO}(2)_{\mathrm{D}} \,,$$

where "ND" denotes ND directions on D5-brane worldvolume and "ND$'$" denotes ND directions on D$5'$-brane worldvolume. Together with 4d R-symmetry, the total symmetry now is

$$\mathrm{SO}(1,3) \times \mathrm{U}(1)_{\mathrm{ND}} \times \mathrm{U}(1)_{\mathrm{ND}'} \times \mathrm{U}(1)_{\mathrm{D}} \times \mathrm{U}(1)_o \,.$$

The representations of various massless field are as follows,

- Six-dimensional vector multiplet generated by 5-5 string:

$$\begin{aligned}
&V_M\colon\ (4,0,0,0,0) \oplus (1,\pm,0,0,0),\\
&\phi^A\colon\ (1,0,\pm,0,0) \oplus (1,0,0,\pm,0),\\
&\psi^+\colon\ (2,+,+,+,0) \oplus (2,+,-,-,0) \oplus (\bar{2},-,+,+,0) \oplus (\bar{2},-,-,-,0),\\
&\psi^-\colon\ (2,-,+,-,0) \oplus (2,-,-,+,0) \oplus (\bar{2},+,+,-,0) \oplus (\bar{2},+,-,+,0).
\end{aligned}$$

- Four-dimensional hypermultiplet generated by 5-$5'$ string:

$$\begin{aligned}
&\text{Scalars } \varphi:\ (1,+,+,0,\pm) \oplus (1,-,-,0,\pm),\\
&\text{Spinors } \lambda\colon\ (0,0,+,\pm) \times |s_0, \tfrac{1}{2}\rangle \ \oplus\ (0,0,-,\pm) \times |s_0, -\tfrac{1}{2}\rangle,
\end{aligned}$$

  where $|s_0, s_1\rangle$ denotes spinor in (3+1)-dimension spacetime.

3.2 Twisting on spectrum

Now we consider D5-branes wrapping on a supersymmetric 2-cycle. It twist the above spectrum and consequently some states become massive and decouple from others. As for D5-D9 system, open string charge $\mathrm{U}(1)_o$ plays an important role in twisting the fundamental hypermultiplet. We claim that the following two choices are valid for our purpose.

1) $\mathrm{U}(1)_{\mathrm{ND}} = \mathrm{U}(1)_{\mathrm{D}} = \mathrm{U}(1)_o$. Total charge is defined by $\mathrm{U}(1) = \mathrm{U}(1)_{\mathrm{ND}} \times \mathrm{U}(1)_{\mathrm{D}} \times \mathrm{U}(1)_o$. Then under $\mathrm{SO}(1,3) \times \mathrm{U}(1)$ symmetry various states reduce to

V_M: $4_0 \oplus 1_\pm$,

ϕ^A: $2 \times 1_0 \oplus 1_\pm$,

ψ^+: $2_0 \oplus \bar{2}_0 \oplus 2_{++} \oplus \bar{2}_{--}$,

ψ^-: $2_0 \oplus \bar{2}_0 \oplus 2_{--} \oplus \bar{2}_{++}$,

φ: $2 \times 1_0 \oplus 1_\pm$,

λ: $|s_0, s_1\rangle_0 \oplus |s_0, 1.2\rangle_{++} \oplus |s_0, -1/2\rangle_{--}$,

Therefore, we end with the $D = 4, \mathcal{N} = 1$ SU(N) gauge theory whose massless spectrum includes a vector multiplet, a chiral multiplet in the adjoint representation and M chiral multiplets in the fundamental representation of the gauge group. When D5$'$-branes are absent, this choice reduces to $\mathrm{U}(1)_J = \mathrm{U}(1)_D \in D(\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R)$, and the resulted field theory is $D = 4$, $\mathcal{N} = 2$ pure SYM.

2) $\mathrm{U}(1)^{1/2}_{\mathrm{ND}} = \mathrm{U}(1)_{\mathrm{ND}'} = \mathrm{U}(1)_{\mathrm{D}} = \mathrm{U}(1)_o$. Total charge is defined by $\mathrm{U}(1) = \mathrm{U}(1)_{\mathrm{ND}} \times \mathrm{U}(1)_{\mathrm{ND}'} \times \mathrm{U}(1)_{\mathrm{D}} \times \mathrm{U}(1)^{3/2}_o$. Under $\mathrm{SO}(1,3) \times \mathrm{U}(1)$ symmetry various states reduce to

V_M: $4_0 \oplus 1_\pm$,

ϕ^A: $2 \times 1_+ \oplus 2 \times 1_-$,

ψ^+: $2_0 \oplus \bar{2}_0 \oplus 2_{++} \oplus \bar{2}_{--}$,

ψ^-: $2 \times 2_- \oplus 2 \times \bar{2}_+$,

φ: $2 \times 1_0 \oplus 1_\pm$,

λ: $|s_0, s_1\rangle_0 \oplus |s_0, 1/2\rangle_+ \oplus |s_0, -1/2\rangle_-$,

We again end with the $D = 4, \mathcal{N} = 1$ SU(N) gauge theory but whose massless spectrum now includes a vector multiplet and M chiral multiplets in the fundamental representation of the gauge group. When D5$'$-branes are absent, this choice reduces to $\mathrm{U}(1)_J = \mathrm{U}(1)_L \in \mathrm{SU}(2)_L$, and the resulted field theory is $D = 4$, $\mathcal{N} = 1$ pure SYM.

Most remarks on D5-D9 system in the previous section is still applicable here. The main advantage for D5-D5$'$ system is D5$'$-branes can be treated as a probe in the D5 background without any fine tuning like in a D5-D9 system. Hence at the probe limit the supergravity background on wrapping D5-branes can be used to study the dual gauge theory with fundamental flavors. In particular, supergravity on the MN background is dual to the large N SQCD with (non-chiral) fundamental quarks. The effect of fundamental flavors is reflected by the DBI action on a D5$'$-brane probe. In order to manifest the quantum number of fundamental flavors, NS gauge field should be turned on in supergravity. It corresponds to fluctuation on the MN background and is expected to be of order M/N.

Different aspect arises from considering a 2-cycle (denotes 2$'$-cycle) wrapped by D5$'$-branes. It is orthogonal to the 2-cycle wrapped by D5-branes and inside a CY three-fold. The 2$'$-cycle can be either topological trivial or non-trivial. If it is topological non-trivial, D5$'$-branes will carry induced charge from the D5-brane background. Because D5$'$-branes are not spacetime filling now, there is no dangerous tadpole problem. Meanwhile, D5′-branes are "real" probes on background. They are probe interaction not only of fundamental flavors but also of gauge bosons on D5-branes. If the 2′-cycle is topological trivial, D5′-branes do not carry any induced charge. Then there is no R-R tadpole excitation even for spacetime filling D5′-branes. The stability of D5′-branes is ensured by negative mass modes propogating in 5d spacetime. In addition, absence of R-R tadpole implies that there should be no anomaly in massless spectrum. In other words, anti-fundamental flavors should appear besides of fundamental flavors found previously. This is achieved because although total induced R-R charge carried by D5′-branes vanishes, the R-R potential does not vanish in general. When D5′-branes wrap on topological trivial 2′-cycle inside CY 3-fold, the remainder part of CY space can be represented as product form of two topological equivalent S^2. The R-R field strength carried by D5′-branes can spread along two different transverse directions in order to get vanishing total charge. A special case is that one component spread on a S^2 wrapped by D5-branes, and another component spread on another S^2. The flux consequently splits S^2 into two disconnect pieces since S^2 is compact, i.e., D5-branes are also split into two pieces. The two pieces carry opposite charge since they possess opposite orientation. The open strings stretching between D5′-branes and one piece of D5-branes give massless fundamental flavors, while one stretching between D5′-branes and another piece of D5-branes give massless anti-fundamental flavors. In other words, the flavors are doubled when D5′-branes wrap on topological trivial 2′-cycle.[2] This point will be manifested via geometry analysis in the next subsection.

3.3 Geometry analysis

We are interested in the MN background. At the probe limit the effect of D5′-branes is ignored. The background reads [8]

$$
\begin{aligned}
ds^2 &= e^{\Phi} dx_{1,3}^2 + e^{\Phi}\alpha' g_s N \left[e^{2h}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + d\rho^2 + \sum_{a=1}^{3}(w^a - A^a)^2 \right], \\
F_{(3)} &= 2\alpha' g_s N \prod_{a=1}^{3}(\omega^a - A^a) + \alpha' g_s N \sum_{a=1}^{3} F^a \wedge \omega^a
\end{aligned}
\tag{3.1}
$$

with $F^a = dA^a + \epsilon^{abc} A^b \wedge A^c$,

$$
\begin{aligned}
e^{2h} &= \rho \coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4}\,, \\
e^{2\Phi} &= e^{2k-2h} = c e^{-h} \sinh 2\rho\,, \qquad a = \frac{2\rho}{\sinh 2\rho}\,,
\end{aligned}
\tag{3.2}
$$

where c is an integral constant, w^a parameterize the 3-sphere,

$$
\omega^1 = \frac{1}{2}(\cos\psi d\theta_2 + \sin\psi \sin\theta_2 d\phi_2)
$$

[2] The splitting, however, does not affect gauged vector supermultiplet because that multiplet is supported by flat part of D5-branes, contrary to hypermultiplet which associated to wrapped directions of 5-branes.

$$\begin{aligned}
\omega^2 &= -\frac{1}{2}(\sin\psi d\theta_2 - \cos\psi \sin\theta_2 d\phi_2) \\
\omega^3 &= = \frac{1}{2}(d\psi + \cos\theta_2 d\phi_2)
\end{aligned} \tag{3.3}$$

and gauge field A_a are written as

$$A^1 = -\frac{1}{2}ad\theta_1\,, \qquad A^2 = \frac{1}{2}a\sin\theta_1 d\phi_1\,, \qquad A^3 = -\frac{1}{2}\cos\theta_1 d\phi_1\,. \tag{3.4}$$

Then the D5-brane charge is given by

$$\tau_5 = \int_{S^3} F_{(3)} \sim \alpha' g_s N\,. \tag{3.5}$$

D5-branes wrapping on S^2 is parameterized by $\{\theta_1,\, \phi_1\}$ which shrinks to zero for $\rho \to 0$. Then D5-branes disappear in resolution and fractional D3-branes are created. D5$'$-branes, meanwhile, may wrap on another orthogonal S'^2 parameterized by $\{\theta_2,\, \phi_2\}$ which carries induce charge. This S'^2 is with finite radius for $\rho \to 0$. Consequently no fractional D3$'$-branes are created in resolution. The finite size of S'^2 implies that gauge theory on probing D5$'$-branes flow to a conformal point in IR. Hence effects of fundamental flavors are manifested in IR even at the probe limit. If we go beyond the probe limit, both S^2 wrapped by D5 and D5$'$-branes no longer shrink to zero because there is a new U(1) flux go through them. This U(1) flux, however, does not yield any singularity. It just reflects the effect of fundamental matter in background and shifts charge carried by the D5$'$-brane by M.

Another interesting possibility is that D5$'$-branes wrap on a topological trivial 2-cycle inside a CY three-fold, namely $\{\rho,\, \psi\}$ directions. Thus they are now spacetime filling branes. To introduce a cut-off in UV (large ρ), the energy ratio of D5$'$-branes to a D5-brane probe locating at UV is about $(\ln\ln r_0/l)^{-2}$. It means that D5$'$-brane can be consistently treated as a probe. The induced magnetic and electric charge carried by D5$'$-branes are now

$$\begin{aligned}
\int F'_{(3)} &\sim \alpha' g_s N(\cos\theta_1 - \cos\theta_2)\,, \\
\int *F'_{(3)} &\sim \int(\tan\theta_2 d\theta_2 - \tan\theta_1 d\theta_1) = 0\,.
\end{aligned} \tag{3.6}$$

Charge quantization condition requires

$$\int F'_{(3)} = 0 \quad \Rightarrow \quad \theta_1 = \theta_2\,. \tag{3.7}$$

In other words, D5$'$-branes do not carry any induced charge. The stability of D5$'$-branes is ensured by the mechanism mentioned above.

The eq. (3.6) implies that induced magnetic and electric flux carried by D5$'$-branes do not vanish although total charges vanish. In particular, the electric field strength is proportional to

$$*\,F'_{(3)} \sim dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\rho \wedge d\psi \wedge (\tan\theta_1 d\theta_1 - \tan\theta_2 d\theta_2)\,. \tag{3.8}$$

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It indicates one components of electric flux spread in θ_1 direction which splits S^2 wrapped by D5-branes into two pieces. It is consistent with argument in previous subsection. Consequently, we obtain not only M fundamental flavors, but also M anti-fundamental flavors.

There is an additional remark on spacetime filling D5′-branes because it is similar to the case described by Karch and Katz [3]. The simple induced form of the worldvolume metric on D5′-branes is modified when the mass term for chiral multiplet is turned on. It is achieved by separating D5-branes and D5′-branes with a small distance c in x^9 direction. Using parameterization of eq. (3.3), $x^9 = e^{\Phi/2}\sin\theta_2$ in unit $\sqrt{\alpha' g_s N}$ leads to

$$\theta_2 = \sin^{-1}\left(\frac{ce^{-\Phi(\rho)}}{2}\right) \sim ce^{-\Phi(\rho)/2} + \cdots , \tag{3.9}$$

where on the D5′ worldvolume ρ takes values between ∞ and 0. For $c = 0$ we recover the topological trivial 2′-cycle defined by $\theta_1 = \theta_2 = 0$. Then fluctuation of worldvolume scalar θ_2 corresponds to the mass perturbation of the chiral multiplet. The ingredient different from ref. [3] is that D5′ is till space-time filling instead of ending in the middle of nowhere.

4. Summary and more discussions

We have shown that a few fundamental flavors can be introduced into the closed string dual of large-N non-chiral SQCD by adding a few D9-branes or orthogonal D5′-branes to a wrapped D5-brane background. The twist on the resulted spectrum is much subtler than without probing D-branes. The essential point is that an open string sector is introduced in the string dual of the field theory. Consequently open string charge plays a role in twisting with the closed string charge. All resulted field theory are $D = 4$, $\mathcal{N} = 1$ gauge theory with fundamental chiral multiplet and with (or without) adjoint chiral multiplet. At the probe limit the known supergravity solution needs not to be modified. The effect of fundamental flavors is manifested by the induced DBI action on worldvolume of probing branes. If probing branes do not carry any induced charges, their stability is ensured by the negative mass modes in 5d spacetime. In particular, adding spacetime filling D5′-branes in the MN background makes the background supergravity dual to a large-N SQCD with non-chiral quarks.

We can introduce two distinct sets of probing D5-branes into the original wrapped D5 background. The three sets of D5-branes are orthogonal to each other. Then this configuration gives rise to a $\mathrm{SU}(M_1) \times S(M_2)$ global symmetry, as in QCD. It is a non-chiral global symmetry. When we introduce mass deformation mixing two distinct flavors, for $M_1 = M_2$ the global symmetry in the field theory gets broken to the diagonal one. It corresponds to two orthogonal 5-branes merging into one smooth 5-branes wrapping on a 2-cycle [19, 20]. A unsolved difficulty is how to incorporate chiral symmetry via the above approach. The usual treatment is to use Hanany-Witten setups [21] in brane setups [22]. The setups, however, have to introduce NS5 branes and a background with singularity such as orbifold [23] and consequently it destroies the original wrapped D5-brane background.

We have pointed out that the worldvolume theories of probe branes include effects of fundamental flavor. Authors of refs. [16, 17] argued that, for a spacetime filling brane probe, the worldvolume theory is just a meson theory. The meson masses and mass gap has been studied in the AdS case. In principle it can be extended to the MN background. The difficulty is that the MN background are much more complicated than the AdS background, and there is no explicit R-symmetry to classify the meson spectrum.

A. Harmonic function on a 5-cycle inside a CY 3-fold

A negative mass mode in the AdS$_{d+1}$ space does not lead to instability as long as the mass is above the BF bound [4], $(mR)^2 \geq -d^2/4$ with curvature radius R. In scenario of AdS$_5$/CFT$_4$ correspondence, the negative mass mode is from Kaluza-Klein spectrum of the IIB supergravity on AdS$_5 \times S^5$ [24, 25]. For example, eigenvalues of the scalar harmonic function on S^5 are $-l(l+4)$ and they are exactly related to the spectrum in AdS$_5$ as $(mR)^2 = l(l+4) \geq -4$. In the MN background, the masses of particles propagated in 5d bulk spacetime are also determined by the Kaluza-Klein spectrum of the IIB supergravity on a compact 5-cycle inside a CY 3-fold. In other words, we need to evaluate eigenvalues of harmonic functions on the 5-cycle. Because this 5-cycle possesses normal bundle structure, it is very difficult to evaluate all eigenvalues of harmonic functions. In this paper we only focus on the existence of positive eigenvalues (negative masses). Precisely, there is no usual eigenvalue for harmonic functions on the 5-cycle since metrics on a 5-cycle depends on the radius parameter ρ. Thus we should call them as eigenvalue-function of ρ. It works as a potential in 5d bulk spacetime. We will show, however, there is a constant part in the eigenvalue-function of scalar harmonic function, which generate masses of particles propagated in 5d bulk spacetime.

Let us consider IIB equation of motion for the scalar field H in 10d,

$$\nabla_{10}^2 H = \frac{1}{\sqrt{-\hat{g}}}\partial_M(\hat{g}^{MN}\partial_N H) = \nabla_{bulk}^2 H + \nabla_5^2 H = 0\,, \tag{A.1}$$

with

$$\nabla_{bulk}^2 = \alpha' g_s N \eta^{\mu\nu}\partial_\mu\partial_n u + \frac{e^{-4\phi}}{\sqrt{g_5}}\frac{\partial}{\partial\rho}\left(e^{4\phi}\sqrt{g_5}\frac{\partial}{\partial\rho}\right)\,, \tag{A.2}$$

is the Laplace operator defined by bulk metric in eq. (3.1) and

$$\nabla_5^2 = \frac{1}{\sqrt{g_5}}\partial_i(g^{ij}\sqrt{g_5}\partial_j)\,, \tag{A.3}$$

is defined by 5-cycle metric

$$ds_5^2 = e^{2h}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \sum_{a=1}^{3}(w^a - A^a)^2\,, \tag{A.4}$$

where $g_5 = e^{4h}\sin^2\theta_1\sin^2\theta_2$ is the determinant of this metric.

At UV limit ($\rho \to \infty$) the metric (A.4) reduces to

$$ds_5^2 \simeq \rho(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{4}\left[d\theta_2^2 + \sin^2\theta_2 d\phi_2^2 + (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2\right]. \tag{A.5}$$

It has the product form of $S^2 \times S^3$. Since radius of S^2 goes to infinity, harmonic functions on S^2 give continuous spectrum. The main contribution is from S^3 with constant radius. The eigenvalue of scalar harmonic function on S^3 is $-l(l+2)$. It indeed contains a positive eigenvalue 1.

At IR limit ($\rho \to 0$) the metric (A.5) becomes

$$\begin{aligned} ds_5^2 \simeq\ & \rho^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{4}(d\theta_1 + \cos\psi d\theta_2 + \sin\psi \sin\theta_2 d\phi_2)^2 + \\ & + \frac{1}{4}(\sin\psi d\theta_2 - \cos\psi \sin\theta_2 d\phi_2 + \sin\theta_1 d\phi_1)^2 + \\ & + \frac{1}{4} d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 \,. \end{aligned} \tag{A.6}$$

At $\rho = 0$ the S^2 shrinks to zero and it is S^3 with constant radius. At the leading order of ρ^2 expansion, the explicit form of the Laplace operator on 5-cycle is

$$\begin{aligned} \rho^2\nabla_5^2 \simeq\ & \frac{1}{\sin\theta_1}\frac{\partial}{\partial\theta_1}\left(\sin\theta_1 \frac{\partial}{\partial\theta_1}\right) + (\cot^2\theta_1 + \cot\theta_1\cot\theta_2\cos\psi)\frac{\partial^2}{\partial\psi^2} - \cos\psi\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\theta_2} + \\ & + 2\sin\psi\cot\theta_2 \frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\psi} + \frac{1}{\sin^2\theta_1}\frac{\partial^2}{\partial\phi_1^2} - \frac{2\sin\psi}{\sin\theta_2}\frac{\partial}{\partial\theta_1}\frac{\partial}{\partial\phi_2} - \\ & - \frac{2}{\sin\theta_1}(\cot\theta_1 + \cos\psi\cot\theta_2)\frac{\partial}{\partial\psi}\frac{\partial}{\partial\phi_1} + \frac{\cos\psi}{\sin\theta_1\sin\theta_2}\frac{\partial}{\partial\phi_1}\frac{\partial}{\partial\phi_1} + \\ & + (\theta_1 \leftrightarrow \theta_2,\ \phi_1 \leftrightarrow \phi_2) + O(\rho^2)\,. \end{aligned} \tag{A.7}$$

Here we focus on the maximal non-zero eigenvalue of the above operator. Two quantum number m_1 and m_2 are generated by two U(1) symmetries associating to the shift of ϕ_1 and ϕ_2. The maximal eigenvalue of the Laplace corresponds to $m_1 = m_2 = 0$. Then harmonic function $H = \sin\theta_1 \sin\theta_2 \sin\psi$ gives maximal eigenvalue -2. Hence ∇_5^2 at the leading order we yield an attractive potential

$$\nabla_5^2 \sim -\frac{\lambda}{\rho^2}\,, \qquad \lambda = 0, 2, \ldots\,. \tag{A.8}$$

It is obtained by shrinking S^2 wrapped by D5-branes to zero.

The negative mass mode in 5d bulk spacetime comes from the sub-leading order of ∇_5^2 whose eigenvalues are constant. In fact, explicit expression of ∇_5^2 in sub-leading order is precisely the same as the Laplace on S^3. It agrees with our first glimpse that the 5-cycle is equivalent to S^3 at the IR limit. The eigenvalue of scalar harmonic function on a 5-cycle, therefore, indeed contains a positive constant eigenvalue 1 at the sub-leading order.

Matching UV result with IR result, we conclude that eigenvalue-function of scalar harmonic function has the form

$$-\,l(l+2) + V(\rho)\,, \tag{A.9}$$

where

$$\begin{aligned} V(\rho) &= \frac{a_1}{\rho} + \frac{a_2}{\rho^2} + \cdots\,, \quad \rho > 1\,, \\ V(\rho) &= \frac{b_1}{\rho^2} + b_2\rho^2 + \cdots\,, \quad \rho < 1\,. \end{aligned} \tag{A.10}$$

This result indicates that there are indeed negative mass modes in the 5d bulk parameterized by the MN background. They associate to the Kaluza-Klein spectrum on S^3 in a 5-cycle. When we introduce spacetime filling D-branes probe in the MN background, they cancels the NS-NS tadpole and consequently it ensure the stability of spacetime filling branes.

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